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Abstract

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Keywords

Asphalt Mixture, Coarse-Graded Mixture, Field Performance

Disciplines

Structural Engineering | Transportation Engineering

Comments

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Performance Evaluation of Coarse-Graded Field Mixtures using Dynamic Modulus Results Gained from Testing in Indirect Tension Mode

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Abstract

Historically, asphalt mixtures in Minnesota have been produced with fine gradations. However, recently more coarse-graded mixtures are being produced as they require less asphalt binder. Thus, it is important that pavement performance for coarse gradations be evaluated. Within this research work, performance evaluation took place with the use of the Dynamic Modulus Test in Indirect Tension mode on coarse-graded mixtures consisting of field cores from 9 different pavements located in five districts of Minnesota. From each pavement's surface layer, 3 specimens were tested at three temperatures; 0.4°C, 17.1°C, and 33.8°C each at nine frequencies ranging between 0.1 Hz and 25 Hz. Additional volumetric characterization of the field mixtures was done to determine asphalt content, air voids, and blended aggregate gradations. Asphalt binders were extracted and recovered for use in determining binder shear complex master curves. Through this information the modified Witczak model was used to create $|E^*|$ master curves which were then compared against the indirect tension (IDT) test $|E^*|$ experimentally created master curves. From the results the Modified Witczak Model needs to be modified for IDT collected dynamic modulus data.

Keyword: Asphalt Mixture, Coarse-Graded Mixture, Field Performance

INTRODUCTION

Understanding of the stress-strain behavior of pavement materials under repetitive traffic loading is necessary to predict the pavement performance and service life. The dynamic modulus test is accepted by pavement agencies as a critical parameter for pavement design and a dynamic modulus master curve for asphalt concrete is an important input for flexible pavement design in the mechanistic-empirical pavement design guide developed in NCHRP Project 1-37A (Kim, et al., 2004). In this research this property was chosen to determine material stiffness and understand its behavior according to temperature (environment) and time of loading. For this work three replicate specimens are tested at three temperatures (0.4°C, 17.1°C, and 33.8°C) and nine frequencies between 25 Hz and 0.1 Hz. The master curves and shift factors are then developed from this database using numerical optimization.

One of the issues related to the role of the dynamic modulus in pavement management is its use in forensic studies and pavement rehabilitation design. It is often impossible to obtain 4-inch (101.6 mm) diameter and 6-inch (152.4 mm) tall asphalt concrete specimens from individual pavement layers for use in dynamic modulus testing, because many asphalt layers are less than a few inches thick. Therefore, the indirect tension (IDT) mode testing of field cores seems to be more appropriate for the evaluation of dynamic modulus. In forensic studies another challenge is designing asphalt mixes in a multi-layered system. These layers have different aggregate gradation and binder content, and stiffness, so they commonly have different dynamic modulus values. In the uniaxial dynamic modulus test this difference is often not considered, but it is possible to measure a layers' dynamic modulus values separately using the IDT mode and create master curves for each layer. To use dynamic modulus prediction models, volumetrics and binder results such as G^* are invaluable. Another focus of this work is to evaluate whether the Modified Witczak model compares well against the experimentally shifted dynamic results (Bari and Witczak, 2006).

OBJECTIVES

The objectives of this research are: 1) performance evaluation of coarse graded pavement using dynamic modulus in IDT mode and 2) comparison between laboratory data and Modified Witczak Model outputs for dynamic modulus in the IDT mode.

MATERIAL AND METHODS

Materials

Within this research work, performance evaluation took place on coarse-graded field cores from 9 different pavements located in five districts of Minnesota as shown in Figure 1 and Table 1. From each pavement's surface layer, 3 specimens were used for testing.

Group 1: K-TH220	(District 2)
Group 2: L-CSAH10	(District 1)
Group 3: M, N-TH27	(District 3)
Group 4: O, P-TH9	(District 2)
Group 5: Q, R-TH28	(District 4)
Group 6: S-TH6	(District 2)
Group 7: T- TH10	(District 4)
Group 8: U-CSAH30	(Metro)
Group 9: W, V-TH10	(District3)

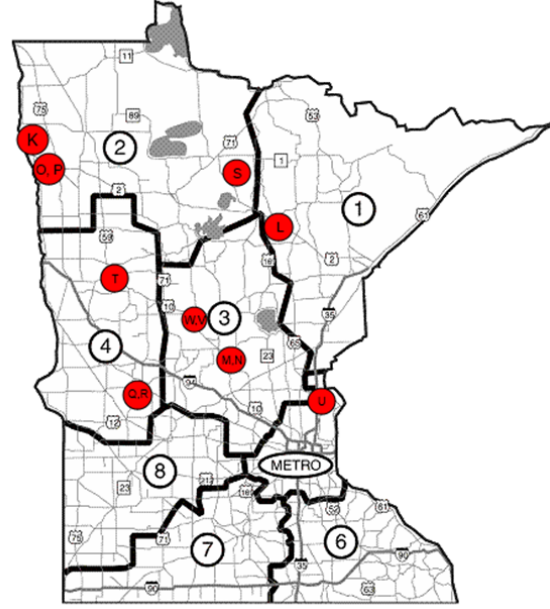


Figure 1: Locations of Pavement Sections in Minnesota

Table 1: Pavement Section Information

Section	MnDOT District	Construction Year	Specimen Letter	Group No.	Construction Type
TH 220	2	2012	K	1	3" M/O
CSAH 10	1	2012	L	2	1.5" O/L on old AC
TH27	3	2010	M, N	3	3" M/O
TH 9	2	2011	O, P	4	3" O/L on reclaimed AC
TH 28	4	2012	Q, R	5	4.5" M/O
TH 6	2	2010	S	6	1.5" M/O
TH 10	4	2013	T	7	3.5" M/O
CSAH 30	Metro	2012	U	8	6" M/O
TH 10	3	2005	V	9	4" M/O (sealed cracks)
TH 10	3	2005	W	9	4" M/O (cracks not sealed)

Note: M/O = Mill and Overlay; O/L = Overlay

Methods

Dynamic Modulus Testing

The complex dynamic modulus $|E^*|$ is a complex number that describes the relationship between stress and strain for a linear viscoelastic material under sinusoidal loading, and it is defined as the ratio of amplitude of the sinusoidal stress and sinusoidal strain in a steady state response as shown in Equation 1 (Dogan, et al., 2003, Schwartz, 2005).

$$E^* = \frac{\sigma}{\varepsilon} = \frac{\sigma_0 \cdot e^{i\omega t}}{\varepsilon_0 \cdot e^{i(\omega t - \delta)}} = \frac{\sigma_0 \cdot \sin(\omega t)}{\varepsilon_0 \cdot \sin(\omega t - \delta)} \quad (1)$$

Where E^* = complex modulus; σ_0 = peak (maximum) stress; ε_0 = peak (maximum) strain; δ = phase angle, degrees; ω = angular velocity; t = time, seconds; e = exponential; and i = imaginary component of the complex modulus. Thus, the dynamic modulus is defined as:

$$|E^*| = \frac{\sigma_0}{\varepsilon_0} \quad (2)$$

The dynamic modulus is a performance related property that can be used for mixture evaluation and characterizing the stiffness of hot mix asphalt (HMA) for use in mechanistic-empirical pavement design. The indirect tension (IDT) mode dynamic modulus test protocol was evaluated by Kim (Kim, et al., 2004) using 6-inch (152.4 mm) diameter, 1.5-inch (38.1 mm) thick specimens cut from Superpave gyratory compacted (SGC) specimens. Sinusoidal loading is applied in controlled stress mode. Horizontal and vertical deformations are measured from two loose core type miniature linear variable differential transformer (LVDT)s with a 50.8mm gauge length located on each side of a specimen's face. Based on the AASHTO TP 62-07 specification, testing must take place on at least two replicate specimens at five temperatures between 14°F and 130°F (-10°C and 54.4°C) and six loading rates between 0.1 and 25 Hz. In order to remain linear viscoelastic, Kim presented a linear viscoelastic solution and calculated coefficient for Poisson's ratio and dynamic modulus for different specimen diameters and gauge lengths. Based on these results the target horizontal tensile strain is 40-60 microstrains and the target vertical compressive strain should be under 100 microstrains (Kim et, al. 2004). Due to the number of temperatures this specification is more time consuming and costly.

In a recent study Li and Williams found that five test temperatures are not necessary to build an accurate, and smooth master curve. From this study it was found that with three temperatures and nine frequencies an equivalent master curve comparable to one made using results from testing at five temperatures and six frequencies could be developed (Li and Williams, 2012).

One of the important parameters used in Mechanistic-Empirical pavement design for asphalt concrete is the dynamic modulus. This property represents the temperature and frequency-dependent or time-dependent stiffness characteristic of the pavement material. It is used in the AASHTOWare Pavement ME to determine the temperature and rate-dependent behavior of an asphalt concrete layer. For this study, the dynamic modulus test in IDT mode will be performed at three temperatures (0.4°C, 17.1°C, and 33.8°C each at nine frequencies (25, 20, 10, 5, 2, 1, 0.5, 0.2, and 0.1 Hz).

Modified Witczak Model

The modified Witczak model is a semi-empirical method used for asphalt concrete dynamic modulus estimation. It is based on nonlinear regression and was formulated through historical data taken from 346 mixtures (7,400 data points). This model was made in response to the limitations identified by the original Witczak model (Bari and Witczak, 2006, Witczak, et al., 1999). A main limitation of the original Witczak model was its dependence on needing other models to convert binder complex shear modulus values into binder viscosity. Furthermore, the original model was not sensitive to changes in volumetrics such as voids in the mineral aggregate (VMA), voids filled with asphalt (VFA), binder content and air voids. Some of these limitations are addressed in the modified model through use of the following parameters: V_a = percentage of air voids (by volume of mix), V_{beff} = percentage of effective binder content (by volume of mix), $|G_b^*|$ = complex shear modulus of binder (psi), and δ_b = phase angle of binder associated with $|G_b^*|$ (degrees). The modified Witczak model is shown below in Equation 3 (Bari and Witczak, 2006).

$$\log_{10} |E^*| = -0.349 + 0.754(|G_b^*|^{-0.0052}) \times \left(6.65 - 0.032\rho_{200} + 0.0027\rho_{200}^2 + 0.011\rho_4 - 0.0001\rho_4^2 + 0.006\rho_{38} - 0.00014\rho_{38}^2 - 0.08V_a - 1.06\left(\frac{V_{beff}}{V_a + V_{beff}}\right) \right) + \frac{2.56 + 0.03V_a + 0.71\left(\frac{V_{beff}}{V_a + V_{beff}}\right) + 0.012\rho_{38} - 0.0001\rho_{38}^2 - 0.01\rho_{34}}{1 + e^{(-0.7814 - 0.5785 \log|G_b^*| + 0.8834 \log\delta_b)}} \quad (3)$$

Where, $|E^*|$ = dynamic modulus (psi), ρ_{200} = percentage of aggregate passing no. 200 sieve, ρ_4 = percentage of aggregate retained on no.4 sieve, $\rho_{3/8}$ = percentage of aggregate retained on no.3/8" sieve, and $\rho_{3/4}$ = percentage of aggregates retained on no.3/4" sieve.

As part of this study the E^* values are predicted using G_b^* values, and volumetrics using the Modified Witczak Model. As such, the predicted E^* values will be compared with laboratory results to see how well the Modified Witczak Model compares against experimental data gained in the IDT mode for dynamic modulus testing.

DISCUSSION OF RESULTS & ANALYSIS

Before binder extraction and recovery was done, volumetrics in conjunction with testing was completed. After testing was finished, binder was extracted and recovered from one specimen of each group. The binder content was determined based on the amount of binder extracted and recovered and the amount of additional binder via an NCAT ignition oven. The recovered aggregate from each group were then sieved according to AASHTO C136/C136M-14. Other properties such as % V_{beff} , %VMA, %VFA, G_{mm} , and air voids were determined from previous work done on the cores collected at the same time from the same section of roadways as the ones used in this study (Helmer, 2015). This information is shown in Table 2.

Table 2: Sieve analysis results and mix properties.

	Group No.	1	2	3	4	5	6	7	8	9
Sieve Size (%) passing	3/4"	100	100	100	100	100	100	100	100	100
	1/2"	93.9	96.4	87.2	93.5	95.1	96.4	94.1	94.4	94.2
	3/8"	77.5	84.6	73.7	76.4	83.1	87.3	83.4	82	80.9
	#4	49.8	53.1	48.4	52.2	52.2	60.9	63.8	48.2	58.6
	#8	34.4	38.4	35.1	43.6	38.8	46.9	47.1	34.9	46.0
	#30	16.7	18.7	17.9	20.9	18.8	23.4	21.7	19.2	25.9
	#50	10.3	10.8	10.9	11.4	9.9	12.4	11.9	11.8	13.8
	#100	6.1	5.9	6.4	5.8	5.4	6.1	6.6	6.1	7.2
	#200	3.6	3.3	6.2	3.3	3.5	3.4	4.0	3.1	4.0
Mix Property	Group No.	1	2	3	4	5	6	7	8	9
	% RAP	23.8	23.3	37.2	26.2	23.8	36.4	23.3	11.4	45.3
	% AC	4.5	5.2	5.6	4.8	4.8	4.9	5.6	5.3	5.0
	% V_{beff}	4.2	4.1	4.1	3.9	3.5	4.3	4.2	4.0	4.6
	% VMA	13.5	13.5	13.6	13.1	12.5	13.9	13.7	13.4	14.4
	% VFA	70.3	70.4	70.6	69.6	68.1	71.2	70.8	70.2	72.3
	G_{mb}	2.315	2.315	2.315	2.315	2.315	2.315	2.315	2.315	2.315
	G_{mm}	2.406	2.458	2.510	2.479	2.635	2.458	2.479	2.510	2.437
	% V_A	4.010	3.996	3.998	3.982	3.988	4.003	4.000	3.993	3.989

Master curves were developed for G_b^* using the sigmoidal model. The model coefficients and shift factors for each group are shown in Table 3. These values can be used to reconstruct the curves.

Table 3: G^* sigmoidal model coefficients and shift factors.

Group No.	δ	α	β	γ	a	b	c
1	-6.253	12.421	-0.942	0.248	0.000339	-0.113	1.825
2	-5.861	11.656	-0.949	0.257	0.000497	-0.122	1.944
3	-6.292	12.407	-0.973	0.245	0.000404	-0.118	1.906
4	-6.106	12.508	-0.836	0.241	0.000292	-0.103	1.680
5	-6.103	12.530	-0.766	0.253	0.000440	-0.115	1.840
6	-6.190	12.467	-0.979	0.254	0.000372	-0.118	1.906
7	-6.284	12.400	-0.881	0.247	0.000312	-0.105	1.710
8	-6.178	12.490	-0.792	0.250	0.000443	-0.115	1.834
9	-6.271	12.410	-0.900	0.260	0.000362	-0.111	1.800

For comparison purposes $|G_b^*|$ from lab was plotted against the predicted $|G_b^*|$ results using the sigmoidal model for each of the nine different groups as shown in Figure 2 (a). Figure 2 (b) displays an overall comparison of all the results of the groups for the

lab $|G_b^*|$ versus sigmoidal predicted $|G_b^*|$. Table 4 shows the R^2 and correlation coefficient (R) values calculated from fitting lab $|G_b^*|$ values against $|G_b^*|$ predicted by sigmoidal model.

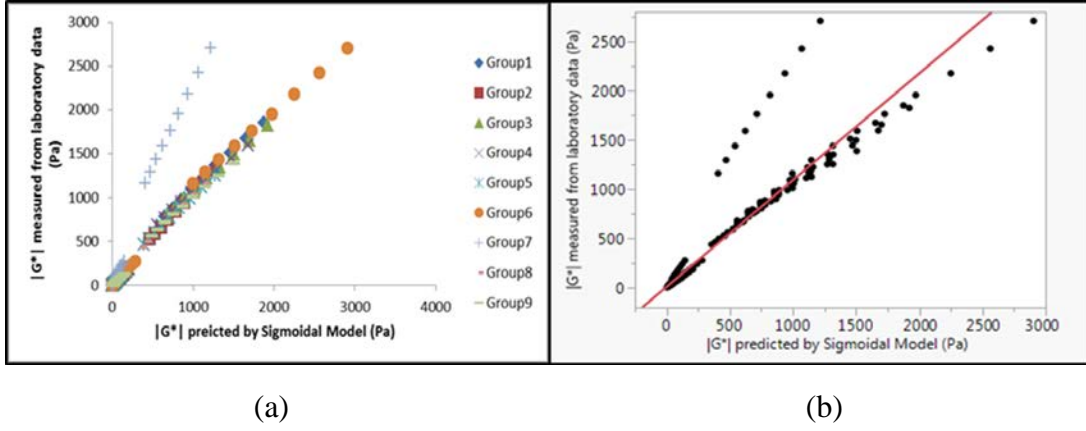


Figure 2: Laboratory Data of $|G_b^*|$ vs. Predicted $|G_b^*|$ (a) each group, (b) all groups together.

Table 4: R^2 and R from fitting lab $|G_b^*|$ values against sigmoidal predicted $|G_b^*|$.

Group No.	1	2	3	4	5	6	7	8	9	All Groups
R^2	0.995	0.998	0.995	0.994	0.996	0.995	0.993	0.997	0.994	0.91
Correlation Coefficient	0.998	0.999	0.998	0.997	0.998	0.998	0.997	0.997	0.997	0.954

From the results shown, the sigmoidal model shows an extremely good fit for the experimental data of each group as well as the data from all the groups put together. This is apparent as both R and R^2 are in the range of 0.91 to 0.999. Dynamic modulus master curves were developed for E^* using the sigmoidal model as well. The model coefficients and shift factors for each group's model are shown in Table 5.

Table 5: E^* sigmoidal model coefficients and shift factors.

Group No.	δ	a	β	γ	a	b	c
1	1.617	2.778	-1.32	0.626	2.171	0.000	-1.849
2	2.363	1.938	-0.762	0.699	2.472	0.000	-2.11
3	2.073	2.288	-1.618	0.659	1.546	0.000	-2.312
4	1.957	2.465	-1.128	0.625	1.734	0.000	-1.828
5	2.51	1.855	-0.31	0.75	1.947	0.000	-1.698
6	1.145	3.301	-1.589	0.565	2.104	0.000	-1.932
7	1.274	3.217	-1.267	0.488	3.267	0.000	-2.199
8	0.79	4.046	-0.985	0.353	2.375	0.000	-1.715
9	1.345	3.041	-1.254	0.638	2.089	0.000	-1.453

To compare the sigmoidal model with the experimentally gained dynamic modulus values shifted to reduce frequencies, Figure 3 was created. Figure 3 is split into two parts (a) separated groups, and (b) all groups data pooled together. From the plots it appears that the sigmoidal model does a very good job fitting the experimental results. The R^2 and R values were determined for each group and for all data from all groups pooled together with results shown in Table 6.

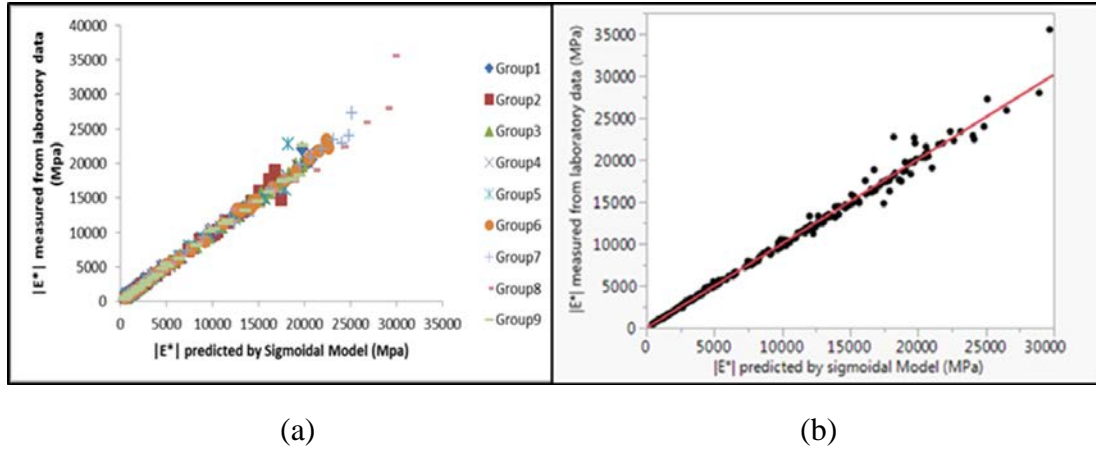
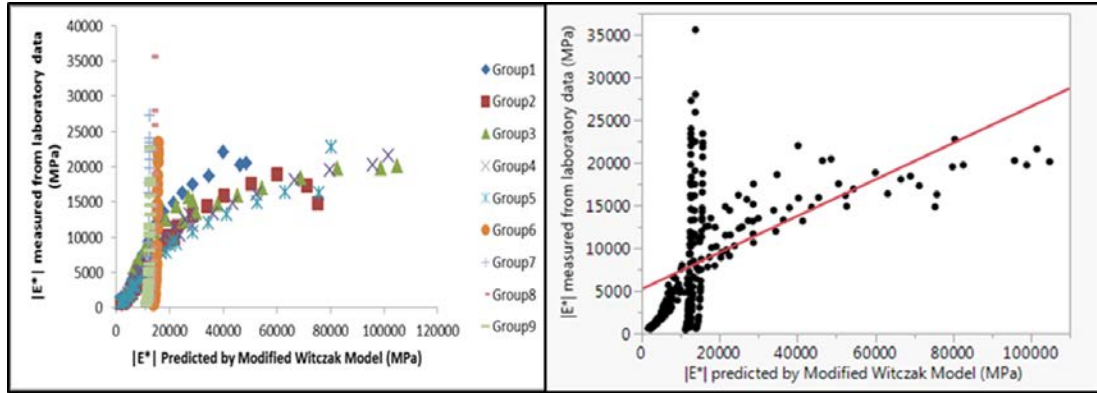


Figure 3: Laboratory Data for $|E^*|$ vs. Predicted $|E^*|$ (a) each group, (b) all groups together.

Table 6: R^2 and R from fitting lab $|E^*|$ values against sigmoidal predicted $|E^*|$.

Group No.	1	2	3	4	5	6	7	8	9	All Groups
R^2	0.995	0.984	0.998	0.998	0.972	0.998	0.995	0.980	0.990	0.990
Correlation Coefficient	0.998	0.992	0.999	0.999	0.990	0.999	0.998	0.990	0.990	0.995

The sigmoidal model shows very good agreement with the experimentally shifted $|E^*|$ results from both Figure 3 (a) and (b) as both the R and R^2 values are in the range of 0.972 to 0.999. Using the $|G_b^*|$ master curve results in combination with volumetrics shown in Table 2, the dynamic modulus master curves were developed using the Modified Witczak Model. Comparison between the experimentally shifted data and Modified Witczak Model predicted data were made for each group and for all the groups pooled together. The results are presented in Figure 4 parts (a) and (b). From the results it is fairly clear that the Modified Witczak Model predicted results do not fit well with the experimentally shifted results for all the groups together as shown in Figure 4 (b). However, it is not clear from visual inspection if the Modified Witczak Model fits well or poorly with the experimentally shifted data for each individual group (Figure 4 (a)). To better examine the best fit models, the R and R^2 values were determined for each group and the all the groups data pooled together as shown in Table 7.



(a)

(b)

Figure 4: Laboratory Data for $|E^*|$ vs. Predicted $|E^*|$ (a) each group, (b) all groups together.

Table7: R^2 and R for lab $|E^*|$ vs. $|E^*|$ predicted values by Modified Witzak model.

Group No.	1	2	3	4	5	6	7	8	9	All Groups
R^2	0.89	0.82	0.74	0.87	0.93	0.79	0.78	0.76	0.77	0.30
Correlation Coefficient	0.94	0.91	0.86	0.93	0.96	0.88	0.88	0.87	0.88	0.54

From the results shown, the Modified Witzak Model works fairly well for each group individually as the R and R^2 range from 0.74 to 0.96. However, looking at the overall fit of all the data together the R and R^2 are 0.54 and 0.30. Examining the fitted plots in Figure 4 does not explain what is happening, so Figure 5 is shown to illustrate why the R and R^2 could be low for the overall fit of all data. Figure 5 shows a comparison between the sigmoidal model and Modified Witzak Model against experimentally shifted data for different groups.

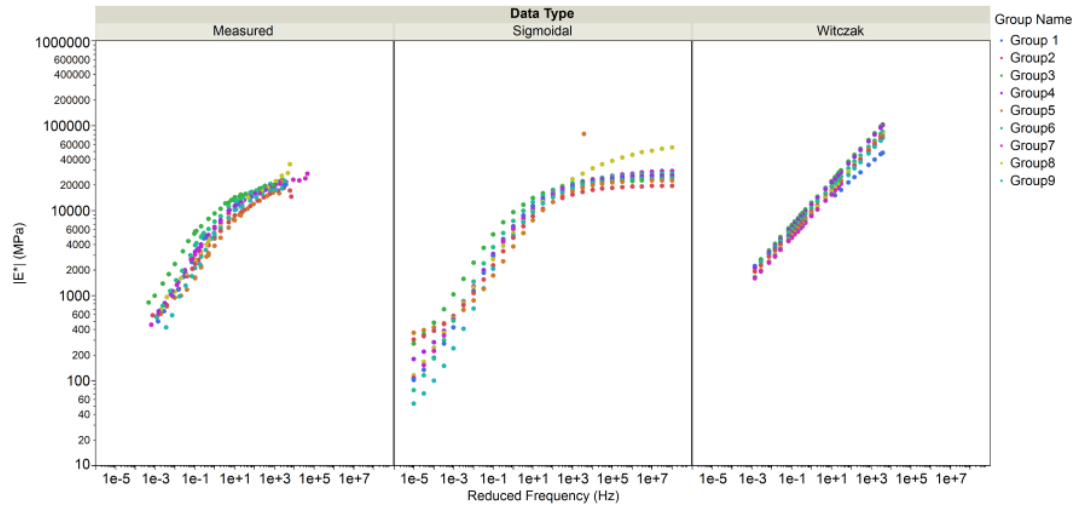


Figure 5: $|E^*|$ Curves for shifted Measured Data, Sigmoidal, and Modified Witczak (Witczak) fitted models.

From the resulting master curves shown in Figure 5 it can be seen that the Modified Witczak Model over estimates the dynamic modulus values from low to high frequencies. This is most likely due to the Modified Witczak Model creation based on historical data gained from testing 4-inch diameter by 6-inch high dynamic modulus specimens.

CONCLUSION

The IDT dynamic modulus test results showed that all nine mix groups have very high stiffness values. The R^2 and R values gained from fitting experimental results against predicted data using the sigmoidal model were close to 1, and thus means the sigmoidal model can be developed and used to predict both $|E^*|$ and $|G_b^*|$ values very well. For the IDT mode of testing, although the Modified Wiczak model can predict $|E^*|$ values using $|G_b^*|$ and other inputs for the more commonly used uniaxial test configuration for determining dynamic modulus values, it is not as accurate in predicting IDT $|E^*|$ values as the sigmoidal model. Due to the ability of IDT dynamic modulus test to more accurately measure the dynamic modulus in asphalt concrete layers collected from field cores, the Modified Witczak Model should be modified for IDT mode in future studies.

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